Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$\therefore$ vol. of circular lamina $aR=\pi(a+c\cos\theta)^2\, c\cos\theta d\theta$ and moment of Inertia about axis through $O$ perpendicular to plane

$$OPx=I=\frac{\pi}{2}(a+c\cos\theta)^4\, c\cos\theta d\theta.$$

Similarly, moment of Inertia of lamina $bR=I'$

$$=\frac{\pi}{2}(a-c\cos\theta)^4\, c\cos\theta d\theta.$$

$\therefore$ moment of Inertia of lamina $ab=I-I'=I''=4\pi ac^2(a^2\cos^2\theta d\theta+c^2\cos^4\theta d\theta)$.

$$\text{But} \int_0^\pi \cos^2\theta d\theta=\frac{\pi}{2};\ \text{and} \int_0^\pi \cos^4\theta d\theta=\tfrac{3}{4}\int_0^\pi \cos^2\theta d\theta=\frac{3\pi}{8}.$$

$\therefore$ $I''=\frac{\pi^2ac^2}{4}(8a^2+6c^2)^4$. The moment of Inertia about an axis through $O$ and in the plane $OPx$ is of course one half of the above amount, or $\frac{\pi^2ac^2}{4}(4a^2+3c^2)$.

[NOTE.—An excellent solution of this problem was received from Professor Wiggins, Richmond, Indiana, but as it has been lost we are unable to publish it. —Editor.]

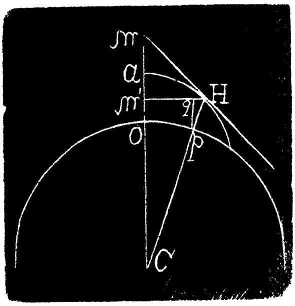
**15. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

Show that the *eastward deviation* of bodies falling from a great height is

$$E_d=\frac{4\pi t(H-\frac{1}{2}\triangle)\cos\phi}{3T}.$$

### Solution by the PROPOSER.

Let $r$=the radius of the earth, $T$=86164.2=the *mean-time* seconds in a siderial day, $\phi$=the observer's latitude, $(r+H)$=the distance from the center of the earth to the point from which the body fell, and $t$=the time of the body's motion in seconds; then the *horizontal velocity* of the falling body in the direction of the tangent to the circle of latitude at the place of observation, is $V=\frac{2\pi(r+H)\cos\phi}{T}$ ....(1); and this velocity is caused by the rotation of the earth. Obviously the *horizontal space* described by the body in the time $t$, is $S_1=Vt=\frac{2\pi t(r+H)\cos\phi}{T}$....(2).



The point on the earth and vertically below the point from which the body fell, passes during this time $t$ through an arc

$$S_2=\frac{2\pi tr\cos\phi}{T}....(3).$$

Hence, the eastward deviation of the falling body, if *the direction of gravity remained parallel toward this body*, would become

$E_a = S_1 - S_2 = \frac{2\pi t H \cos\phi}{T} \ldots (4)$; although this *change of direction* is not great, yet it must not by any means be neglected—even though the variation in the *intensity* of gravity is frequently neglected. Suppose the body in its descent to have arrived at $H$, and let $m'H = x$. Let $w$ represent the body's velocity in the direction $m'H$, and $v$ that in the direction $HC$; then $dv = g dt \ldots (5)$.

Considering $HC = r$, we have the proportion,

$$-dw : dv :: Hq : Hp :: m'H : HC :: x : r.$$

$\therefore \quad dw = -\frac{x}{r} dv, = -\frac{gx}{r} dt \ldots (6)$, the negative sign showing that $w$ *decreases* as $r$ *increases*. Since $w = dx \,/\, dt$, (6) becomes

$$dw = -\frac{gxdx}{rw}. \quad \therefore \; wdw = -\frac{g}{r} xdx \ldots (7).$$

Integrating (7) and remembering that when $x = 0$ the initial value of $w$ is $V$ as given in (1), we have $w^2 = V^2 - \frac{gx^2}{r}$. $\therefore \; w = V\sqrt{\left(1 - \frac{gx^2}{rV^2}\right)} \ldots (8)$.

With $w = dx \,/\, dt$, we now easily deduce

$$Vdt = \frac{dx}{\sqrt{\left(1 - \frac{gx^2}{rV^2}\right)}} = \Big(1 + \frac{1}{2} \cdot \frac{gx^2}{rV^2} + \text{higher powers of } x\text{, which may be neglected.}\Big)dx \ldots (9).$$

Integrating (9), observing that $C = 0$ since $x = 0$ when $t = 0$, and without *appreciable* error putting $x^3 = (Vt)^3$, we have the expression

$$x = Vt - \frac{1}{6} \cdot \frac{gVt^3}{r} \ldots (10).$$

If $\triangle$ represents the *excess of descent* in vacuo above that in air, we have

$$\frac{1}{2} g t^2 = H + \triangle \ldots (11). \quad \therefore \; x = Vt - \frac{(H + \triangle) Vt}{3r} \ldots (12).$$

Transforming (12) by means of (1) and without *sensible* error writing *unity* for $(r + H) \,/\, r$, we have $x = \frac{2\pi t[(r + H) - \frac{1}{3}(H + \triangle)]\cos\phi}{T} \ldots (13)$.

For obvious reasons subtracting (3) from (13), etc., we obtain

$$E_a = \frac{4\pi t(H - \frac{1}{2}\triangle)\cos\phi}{3T} \ldots (14),$$

which is the *expression* for the eastward deviation of bodies falling from a great height, given in *Young's General Astronomy*.

Also solved by *G. B. M. Zerr*. His solution will appear next month.